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# Topological spaces with an $\omega^\omega$-base

*Taras Banakh*

*Jan Kochanowski University in Kielce (Poland) and*

*Ivan Franko National University of Lviv (Ukraine),*

*Kielce-Lviv, Poland-Ukraine*

*t.o.banakh@gmail.com*

**Abstract:** Given a partially ordered set P we shall discuss properties of topological spaces X admitting a P-base, i.e., an indexed family $(U_\alpha)_{\alpha\in P}$ of subsets of $X\times X$ such that $U_\beta \subseteq U_\alpha$ for all $\alpha\le\beta$ in P and for every $x\in X$ the family $(U_\alpha[x])_{\alpha\in P}$ of balls $U_\alpha[x]=\{y\in X:(x,y)\in U_\alpha\}$ is a neighborhood base at x. A P-base $(U_\alpha[x])_{\alpha\in P}$ for X is called *locally uniform* if the family of entourages $(U_\alpha U_\alpha^{-1}U_\alpha)_{\alpha\in P}$ remains a P-base for X. A topological space is first-countable if and only if it has an $\omega$-base. By Moore's Metrization Theorem, a $T_0$-space is metrizable if and only if it has a locally uniform $\omega$-base.

In the talk we shall discuss topological spaces possessing a (locally uniform) $\omega^\omega$-base. Our results show that spaces with an $\omega^\omega$-base share some common properties with first countable spaces, in particular, many known upper bounds on the cardinality of first-countable spaces remain true for countably tight $\omega^\omega$-based topological spaces. On the other hand, topological spaces with a locally uniform $\omega^\omega$-base have many properties, typical for generalized metric spaces. Also we study Tychonoff spaces whose universal (pre- or quasi-) uniformity has an $\omega^\omega$-base and show that such spaces are close to being $\sigma$-compact.

More information can be found in the paper-book [1].

Keywords: generalized metric space, partially ordered set, neighborhood base.

**References:**

[1] T. Banakh, “Topological spaces with an $\omega^\omega$-base”, 105 pp. preprint (https://arxiv.org/abs/1607.07978).